Worksheet 2.10—Derivatives of Log Functions & LOG DIFF

Show all work. No calculator unless otherwise stated.

Short Answer

1. Find the derivative of each function with respect to \( x \), given that \( a \) is a constant
   (a) \( y = x^a \)  
   (b) \( y = a^x \)  
   (c) \( y = x^x \)  
   (d) \( y = a^x \)

2. Evaluate each of the following. Remember to simplify early and often (especially when you have logs).
   (a) \( \frac{d}{dx} \left[ e^{2\ln x} \right] = \)  
   (b) \( \frac{d}{dx} \left[ \log_a a^{\sin x} \right] = \)  
   (c) \( \frac{d}{dx} \left[ \log_2 8^{x-5} \right] = \)

3. For each of the following, find \( \frac{dy}{dx} \). Look to simplify using the properties of logs first.
   (a) \( y = \log_3 \frac{x\sqrt{x}-1}{2} \)  
   (b) \( y = x^{3/2} \log_2 \sqrt{x+1} \)
(c) $y = \ln \left| \frac{\cos x}{\cos x - 1} \right|$

(d) $y = \ln \left( \ln \frac{1}{x} \right)$

(e) $y = \ln^3 x$

(f) $y = x \ln x^2$

(g) $y = \log_3 (1 + x \ln x)$

(h) $y = \ln \sqrt[4]{\frac{4x - 2}{3x + 1}}$
4. Use implicit differentiation to find $\frac{dy}{dx}$.
   
   (a) $x^2 - 3 \ln y + y^2 = 10$
   
   (b) $\ln xy + 5x = 30$

5. Find an equation of the tangent line to the graph of $x + y - 1 = \ln \left( x^2 + y\sqrt{2} \right)$ at $(1, 0)$.

6. A line with slope $m$ passes through the origin and is tangent to  $y = \ln \left( \frac{x}{3} \right)$. What is the value of $m$?
7. Find an equation for a line that is tangent to the graph of \( y = e^x \) and goes through the origin.

8. Find the point where the tangent line to the curve \( y = e^{-x} \) is perpendicular to the line \( 2x - y = 8 \).

9. Use Logarithmic Differentiation to evaluate the following.

(a) \[ \frac{d}{dx} \left[ \sqrt[3]{\frac{(x-3)^4}{(x^2+1)}} \right] = \]

(b) If \( y = x^{1/\ln x} \), find \( \frac{dy}{dx} \).
10. Let \( f(x) = \ln \left( 1 - x^2 \right) \).

   (a) State the domain of \( f \).

   (b) Find \( \lim_{x \to -1^{-}} f(x) \).

   (c) Find \( f'(x) \).

   (d) State the domain of \( f''(x) \).

   (d) Explain why \( f''(x) < 0 \) for all \( x \) in the domain of \( f \).

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Multiple Choice

11. Use the properties of logs to simplify, as much as possible, the expression:

\[ \log_{a} 32 + \frac{4}{5} \log_{a} 4 - \frac{4}{5} \log_{a} 2 + \log_{a} \frac{1}{14} \]

   \( \frac{1}{2^5} \)

   (A) \( \log_{a} 128 \) (B) \( \log_{a} 8 \) (C) \( \log_{a} 32 \) (D) \( \log_{a} 2^{-7} \) (E) 8
____ 12. Simplify the expression: $2^{5(\log_2 e)\ln x}$

(A) $5^x$  
(B) $e^{11}$  
(C) $x^5$  
(D) $x^{10}$  
(E) $x^2$

____ 13. Which of the following is the domain of $f'(x)$ if $f(x) = \log_2 (x+3)$?

(A) $x < -3$  
(B) $x \leq 3$  
(C) $x \neq -3$  
(D) $x > -3$  
(E) $x \geq -3$

____ 14. If $f(x) = (x^2 + 1)^{(2-3x)}$, then $f'(1) =$

(A) $-\frac{1}{2} \ln (8e)$  
(B) $-\ln (8e)$  
(C) $-\frac{3}{2} \ln 2$  
(D) $-\frac{1}{2}$  
(E) $\frac{1}{8}$
15. Determine if \( \lim_{x \to \infty} \left[ \ln (2 + 5x) - \ln (2 + 3x) \right] \) exists, and if it does, find its value.

(A) \( \ln \frac{1}{2} \)
(B) \( \ln \frac{5}{3} \)
(C) \( \ln \frac{3}{5} \)
(D) \( \ln 2 \)
(E) Does Not Exist

16. Find the derivative of \( f(t) = \frac{2 \ln t}{3 + \ln t} \).

(A) \( f'(t) = \frac{2}{t (3 + \ln t)^2} \)
(B) \( f'(t) = \frac{6 \ln t}{(3 + \ln t)^2} \)
(C) \( f'(t) = \frac{6}{(3 + \ln t)^2} \)
(D) \( f'(t) = \frac{2}{t (3 + \ln t)} \)
(E) \( f'(t) = \frac{6}{t (3 + \ln t)^2} \)

17. Determine the derivative of \( f \) when \( f(x) = x^{4x} \)

(A) \( f'(x) = (\ln x + 4)x^{4x} \)
(B) \( f'(x) = 4(\ln x + 1)x^{4x} \)
(C) \( f'(x) = 4(\ln x + 1) \)
(D) \( f'(x) = (\ln x + 1)x^{4x} \)
(E) \( f'(x) = 4x^{4(x-1)} \)

18. Find the derivative of \( f \) when \( f(x) = x\left[ 7\sin (\ln x) + 2\cos (\ln x) \right] \).

(A) \( f'(x) = x\left[ 5\sin (\ln x) + 9\cos (\ln x) \right] \)
(B) \( f'(x) = 5\sin (\ln x) - 9\cos (\ln x) \)
(C) \( f'(x) = 5\sin (\ln x) + 9\cos (\ln x) \)
(D) \( f'(x) = 9\sin (\ln x) + 5\cos (\ln x) \)
(E) \( f'(x) = x\left[ 9\sin (\ln x) + 5\cos (\ln x) \right] \)